

# Moving Domain Walls in $AdS_5$ and Graceful Exit from Inflation

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## Abstract

We consider moving-brane-solutions in AdS type back ground. In the first Randall-Sundrum configuration, there are two branes at fixed points of the orbifold symmetry. We point out that if one brane is fixed and the other brane is moving, the configuration is still a solution provided the moving brane has a specific velocity determined by its tension and bulk cosmological constant. In the absence of the  $\mathbf{Z}_2$  symmetry, we can construct multi-brane configurations by patching AdS-Schwarzschild solutions. In this case, we show that the 4-dimensional effective cosmological constant on the brane world is not well defined. We find a condition for a brane to be stationary. Using the brane scattering, we suggest a scenario of inflation on the brane universe during a finite time, i.e, a scenario of a graceful exit of inflation.

PACS numbers: 12.10.-g, 11.10.Kk, 11.25.Mj

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## I. INTRODUCTION

Recently, Randall and Sundrum made an proposal that we may live in a four dimensional section of 5 dimensional universe [1]. The key point of the first one is that gravitational warp factor can generate a small number to solve the hierarchy problem. The idea of the second one is that due to the the attraction of the brane energy, the metric fluctuation around the domain wall admits a bound state and the background cosmological constant makes this 'bound state' a zero-mode to be identified as a graviton. The brane tension is fine tuned so that the effective cosmological constant on the domain wall is zero and the brane is stationary. If the fine tuning is relaxed, bulk metric depends on time and the brane inflates [2]. These solutions can be used to discuss the cosmology of the RS-model [2,3].

More recently, Kraus pointed out that these solutions can be interpreted as motion of a domain wall in a stationary background [4].<sup>‡</sup> The idea of [4] is that for the given cosmological constant we can glue two pieces of AdS-Schwartzschild (AdSS) solutions along the moving domain wall using the Israel matching conditions to construct the new space-time containing the domain wall. The matching condition determines the motion of the domain wall in terms of the domain wall tension and the bulk cosmological constant.

The moving brane solutions are interesting since those can lead us scenarios for the graceful exit from the inflation in the Randall-Sundrum type cosmology. For example, suppose we live in a brane whose brane tension is fine tuned to have flat 4 dimensional geometry. Now if there is a moving brane with finite thickness, the collision of our brane with such a moving brane provide inflation in our brane world for the finite collision time. Having explicit analytic solution for the two moving branes with 'finite thickness' is not an easy task. So finding solutions for two moving thin branes is the first step in this direction and this is the goal of this paper. Generalization of the RS1 model to time dependent case

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<sup>‡</sup>Ref. [5] discussed the motion of a 'probe brane' in fixed background using the Dirac-Born-Infeld action.

has been discussed in [2] where the distance between the two branes are fixed, which is not appropriate to our purpose.

We first examine the case with orbifold symmetry, which is the generalization of the Randall-Sundrum to the moving brane case. We try to solve the full 5 dimensional bulk Einstein solution with the warp factor corresponding to the moving brane. It turns out that the Einstein equation dictates that the information of the brane location must disappear from the bulk metric. However the motion of the brane is not arbitrary. It is fixed by the tension of the brane and the bulk cosmological constant. We will get a solution with two branes; one is stationary and the other is moving. In the presence of the reflection symmetry, the tensions of the two branes must have opposite sign. To get the solution where both of the branes have positive tension, we have to abandon the orbifold symmetry. We use Kraus' method. We show that the 4-dimensional effective cosmological constant on the brane world in the absence of the reflection symmetry, is not well defined. We find a condition for a brane to be stationary. Finally we consider the brane scattering using these solutions and suggest a scenario for the inflation during finite duration [9].

## II. SUMMARY OF RS-MODEL

We start with following action:

$$S = S_{gravity} + \Sigma_i S_{i(wall)}, \quad (1)$$

where  $S_{gravity}$  is given by

$$S_{gravity} = \int d^5x \sqrt{g} \left[ \frac{1}{2\kappa^2} \mathcal{R} - \Lambda \right], \quad (2)$$

with  $\Lambda$  being a cosmological constant and  $g_{\mu\nu}$  a metric of five-dimensional space-time. The domain wall action can be written as

$$S_{i(wall)} = - \int d^5x \sqrt{g} \mathcal{L}_{i(wall)} \delta(y - R_i(t)), \quad (3)$$

where  $y$  is a coordinate of a transverse direction. Here the delta function implies that the domain wall lies at the position  $y = R_i(t)$  and can possibly move.

Assuming that all excitation modes of the matter on the wall are absent, the action Eq. (1) is reduced to the

$$S = \int d^5x \sqrt{g} \left[ \frac{1}{2\kappa^2} \mathcal{R} - \Lambda \right] - \sum_i \sigma_i \int d^5x \sqrt{g} \delta(y - R_i(t)), \quad (4)$$

where  $\sigma_i$  is a tension of the  $i$ -th domain wall. Since we are interest to the AdS space-time in the bulk, only the case  $\Lambda < 0$  will be considered. From Eq. (4), Einstein equations become

$$\mathcal{R}^\mu{}_\nu - \frac{1}{2} \delta^\mu{}_\nu \mathcal{R} = -\kappa^2 T^\mu{}_\nu. \quad (5)$$

Here, the energy-momentum tensor  $T^\mu{}_\nu$  is given by

$$\begin{aligned} T^\mu{}_\nu &= -|\Lambda| \text{diag}(1, 1, 1, 1, 1) + \Sigma_i T_{i(wall)}^\mu{}_\nu, \\ T_{i(wall)}^\mu{}_\nu &= \sigma_i \text{diag}(1, 1, 1, 1, 0) \delta(y - R_i(t)). \end{aligned} \quad (6)$$

where  $T_{i(wall)}^\mu{}_\nu$  is an energy-momentum tensor on the  $i$ -th wall.

The vacuum solution of the Einstein equation in the presence of the negative cosmological constant  $\Lambda$  is AdS space: The Randall-Sundrum model is constructed by joining two AdS

$$ds^2 = e^{2ky} (-dt^2 + \delta_{ij} dx^i dx^j) + dy^2, \quad (7)$$

where

$$k^2 = \frac{\kappa^2}{6} |\Lambda|, \quad (8)$$

The Randall-Sundrum [1] solution is that domain wall solution can be obtained by joining two 'inner-part' of the AdS spaces,  $y \leq 0$  along the hyper surface at  $y = 0$  [7]:

$$ds^2 = e^{-2k|y|} (-dt^2 + \delta_{ij} dx^i dx^j) + dy^2. \quad (9)$$

The solution has manifest  $\mathbf{Z}_2$  summetry  $y \rightarrow -y$ . The domain wall at  $y = 0$  has tension given by

$$\sigma = 6k/\kappa^2. \quad (10)$$

Now, for later purpose, let's ask what should be the solution if we put the domain wall along  $y = R$ . Due to the  $\mathbf{Z}_2$  symmetry, the metric should be as follows:

$$\begin{aligned} ds^2 &= e^{-2k|y-R|}(-dt^2 + \delta_{ij}dx^i dx^j) + dy^2 \quad \text{for near } y=R, \\ &= e^{-2k|y+R|}(-dt^2 + \delta_{ij}dx^i dx^j) + dy^2 \quad \text{for near } y=-R, \end{aligned} \quad (11)$$

The sign in the warp factors is chosen such that as  $|y| \rightarrow \infty$  we are going deep inside the AdS spaces. Physically, there is no difference whether we have domain wall at  $y = 0$  or  $y = R$  apart from the over all scale  $e^{2kR}$ . This can be verified by observing that above metric can be written as

$$ds^2 = e^{-2k|y|+2kR}(-dt^2 + \delta_{ij}dx^i dx^j) + dy^2, \text{ for } |y| \geq R. \quad (12)$$

Notice that  $y = R$  and  $y = -R$  are identified and there is no domain wall at  $y = 0$  since the region  $-R \leq y \leq R$  is not in the universe in this construction. This is the 1-wall solution with positive tension (RS2) [1].

However, in the region  $-R \leq y \leq R$ , the same expression (11) can be written as

$$ds^2 = e^{2k|y|-2kR}(-dt^2 + \delta_{ij}dx^i dx^j) + dy^2. \quad (13)$$

This means that if  $k$  is positive, there is a domain wall with negative tension at  $y = 0$ , as well as positive tension domain wall at  $y = R$ . Therefore (13) represent solution with two domain walls (RS1). Therefore, according to the region we are looking at, the solution (11) can gives either RS2 or RS1. The RS solution is not a geometry for one ads space with a brane but a patching two ads along a brane [7]. See figure 1(a,b). Notice that the figure 1.(b) also tells us that as  $R \rightarrow \infty$ , RS1 is reduced to RS2. Later on, we will see that replacing  $R \rightarrow R(t)$  is still solution for specific  $R(t)$

If a constant  $R$  is replaced with time dependent  $R(t)$ , we would be considering a moving domain wall. In the next section we are looking for a solution which has a warp factor corresponding moving domain walls.

### III. MOVING DOMAIN WALLS

#### A. RS1-type model

In this section, we look for a solution with moving domain walls located at  $y = R(t)$  and  $y = 0$ . If we impose  $\mathbf{Z}_2$  symmetry, there must be a mirror image of the moving wall at  $y = -R(t)$ . So we are looking for the moving version of the solution (11). Therefore we start from the following ansatz for the region  $-R(t) \leq y \leq R(t)$ :

$$ds^2 = e^{-2k|y|+2kR(t)}(-dt^2 + H(t, y)^2 \delta_{ij} dx^i dx^j) + b(t, y)^2 dy^2, \quad (14)$$

where  $k$  in Eq. (13) is replaced with  $-k$  and this metric is RS1-type metric which has two walls at  $y = 0$  and  $y = R(t)$ . Since we impose  $\mathbf{Z}_2$  symmetry, the above metric must be invariant under  $y \rightarrow -y$ . To determine  $H(t, y)$  and  $b(t, y)$ , we have to solve five-dimensional Einstein equations. If we write the metric as

$$ds_5^2 = -n(t, y)^2 dt^2 + a(t, y)^2 \delta_{ij} dx^i dx^j + b(t, y)^2 dy^2. \quad (15)$$

Einstein equations are given by [2]

$$\begin{aligned} G_{00} &= 3 \left[ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left\{ \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right\} \right] = \kappa^2 T_{00}, \\ G_{ii} &= \frac{a^2}{b^2} \left[ \frac{a'}{a} \left( \frac{a'}{a} + 2 \frac{n'}{n} \right) - \frac{b'}{b} \left( \frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right] \\ &\quad + \frac{a^2}{n^2} \left[ \frac{\dot{a}}{a} \left( -\frac{\dot{a}}{a} + 2 \frac{\dot{n}}{n} \right) + \frac{\dot{b}}{b} \left( -2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - 2 \frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} \right] = \kappa^2 T_{ii}, \\ G_{55} &= 3 \left[ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{b^2}{n^2} \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \frac{\ddot{a}}{a} \right\} \right] = \kappa^2 T_{55}, \\ G_{05} &= 3 \left[ \frac{n'}{n} \frac{\dot{a}}{a} + \frac{a'}{a} \frac{\dot{b}}{b} - \frac{\dot{a}'}{a} \right] = 0, \end{aligned} \quad (16)$$

where we use dot and prime to describe a derivative with respect to  $t$  and  $y$  respectively. Due to  $\mathbf{Z}_2$  symmetry, it is sufficient that we pay attention to the right-hand side of the domain wall only. Our ansatz means that,  $n(t, y)$  and  $a(t, y)$  are given by

$$\begin{aligned} n(t, y) &= e^{-ky+kR(t)}, \\ a(t, y) &= n(t, y)H(t, y) = e^{-ky+kR(t)}H(t, y). \end{aligned} \quad (17)$$

in the region  $0 \leq y \leq R(t)$ . Using these equations,  $G_{05}$  in the bulk reads

$$\left[ \left( \frac{H'}{H} \right) - k \right] \frac{\dot{b}}{b} = \left( \frac{\dot{H}'}{H} \right) + k\dot{R} \left( \frac{H'}{H} \right). \quad (18)$$

Now we take the gauge in which  $b(y, t) = 1$ . Then Eq. (18) can be solved by

$$H(t, y) = e^{-kR(t)} G(y). \quad (19)$$

for any  $G(y)$ . Inserting Eq. (19) into Einstein equations, the other Einstein equations can be rewritten as

$$\begin{aligned} G_{00} &= -3n^2 \left[ \frac{G''}{G} + \left( \frac{G'}{G} \right)^2 - 4k \frac{G'}{G} + 2k^2 \right] = -n^2 \kappa^2 |\Lambda|, \\ G_{ii} &= n^2 H^2 \left[ 2 \frac{G''}{G} + \left( \frac{G'}{G} \right)^2 - 8k \frac{G'}{G} + 6k^2 \right] = n^2 H^2 \kappa^2 |\Lambda|, \\ G_{55} &= 3 \left[ \left( \frac{G'}{G} \right)^2 - 3k \frac{G'}{G} + 2k^2 \right] = \kappa^2 |\Lambda|. \end{aligned} \quad (20)$$

If we set  $G(y) = e^{mky}$ , only the case of  $m = 0$  satisfies all equations in Eq. (20) with following relation:

$$k^2 = \kappa^2 |\Lambda| / 6. \quad (21)$$

Finally, the metric in the region  $0 \leq y \leq R(t)$  is

$$ds^2 = e^{-2ky+2kR(t)} (-dt^2 + e^{-2kR(t)} \delta_{ij} dx^i dx^j) + dy^2. \quad (22)$$

Due to  $\mathbf{Z}_2$  symmetry the metric in the region  $-R(t) \leq y \leq 0$  is

$$ds^2 = e^{2ky+2kR(t)} (-dt^2 + e^{-2kR(t)} \delta_{ij} dx^i dx^j) + dy^2. \quad (23)$$

Therefore the metric in  $-R(t) \leq y \leq R(t)$  can be written simply as

$$ds^2 = e^{-2k|y|} (-e^{2kR(t)} dt^2 + \delta_{ij} dx^i dx^j) + dy^2, \quad (24)$$

which is a AdS metric in the bulk by redefining the time. At  $y = 0$ , there is a static wall. By integrating the Einstein equation across the static wall domain wall, we get the relation of the tension and  $k$ :

$$\sigma = \frac{6k}{\kappa^2} := \sigma_c. \quad (25)$$

This, together with Eq. (21), gives the exact value of the tension of the static wall in terms of the bulk cosmological constant:

$$\sigma = \pm \sqrt{\frac{6|\Lambda|}{\kappa^2}}. \quad (26)$$

where  $\pm = k/|k|$ .

Note that the bulk Einstein equations do not determine  $R(t)$ , the position of the moving wall: for arbitrary function  $R(t)$ , (24) is a solution of the bulk Einstein equations. As is well known,  $R(t)$  is determined by the Israel junction equation [6,4]. In terms of the extrinsic curvature of the wall world-volume

$$K_{\mu\nu} = \nabla_\mu n_\nu, \quad (27)$$

with  $n_\nu$  the unit normal vector on the domain wall world-volume, the Junction condition reads

$$\Delta K_{mn} = -\kappa^2 \left( T_{mn} - \frac{1}{3} T^l_l g_{mn} \right), \quad (28)$$

where  $\Delta K_{mn} := \lim_{y \rightarrow +0} K_{mn} - \lim_{y \rightarrow -0} K_{mn} = K^+_{mn} - K^-_{mn}$ . Here  $g_{mn}$  is a metric on the wall and  $T_{mn}$  is a energy-momentum tensor of the wall. If we impose  $\mathbf{Z}_2$  symmetry,  $\Delta K_{mn}$  is equal to  $2K^+_{mn}$  due to  $K^+_{mn} = -K^-_{mn}$ . For ‘extremal’ wall, the case where  $T_{mn} = \sigma g_{mn}$ , the Israel junction equations become

$$2K^+_{mn} = \frac{\kappa^2}{3} \sigma g_{mn}. \quad (29)$$

To be explicit, let’s write the metric as

$$ds^2 = -d\tau^2 = -f(t, y) dt^2 + g(y) \delta_{ij} dx^i dx^j + dy^2, \quad (30)$$

and let  $u^\mu$  be the tangent velocity vector of the wall satisfying  $u^\mu u_\mu = -1$ . Because the wall moves in  $y$  direction,  $u^\mu$  is given by



$$u^\mu = (\sqrt{\frac{1 + (\partial_\tau y)^2}{f}}, 0, 0, 0, \partial_\tau y). \quad (31)$$

Since the unit normal vector  $n^\mu$  satisfies  $n^\mu u_\mu = 0$ ,

$$n^\mu = (-\frac{\partial_\tau y}{\sqrt{f}}, 0, 0, 0, -\sqrt{1 + (\partial_\tau y)^2}). \quad (32)$$

Insert  $n_\mu$  into Eq. (27), we obtain two junction equations: one is

$$2K^+_{ii} = -\sqrt{1 + (\partial_\tau y)^2} \partial_y g = \frac{\kappa^2}{3} \sigma g, \quad (33)$$

and the other  $K_{00}$  equation does not give an independent one. In our case,  $f(t, y)$  and  $g(t, y)$  are given by  $f(t, y) = e^{2ky+2kR(t)}$  and  $g(t, y) = e^{2ky}$ . The  $ii$  component of Israel junction equations is reduced to

$$\sqrt{1 + (\partial_\tau R(t))^2} = -\frac{\sigma}{\sigma_c}. \quad (34)$$

Notice that if  $k$  is positive, the tension of the moving wall must be negative and  $R(t)$  is determined as

$$R(t) = v\tau, \quad (35)$$

where  $v = \pm \sqrt{|\sigma/\sigma_c|^2 - 1}$ . Using  $u^0 = dt/d\tau = \sqrt{\frac{1 + (\partial_\tau y)^2}{f}}$ ,

$$\tau(t) = \frac{1}{\sqrt{1 + v^2}} t. \quad (36)$$

So  $R(t)$  is given by

$$R(t) = \frac{v}{\sqrt{1 + v^2}} t. \quad (37)$$

From the bulk metric, the four-dimensional metric on the static wall can be reduced to

$$ds_4^2 = -d\tilde{t}^2 + \delta_{ij} dx^i dx^j, \quad (38)$$

where  $d\tilde{t} = e^{kR(t)} dt$ . This metric is a Mikowskian one as is the Randall-Sundrum case. The metric on the moving wall is

$$ds_4^2 = -d\tau^2 + e^{-2kv\tau} \delta_{ij} dx^i dx^j. \quad (39)$$

For  $k > 0$ , the static wall has a positive tension at  $y = 0$  and the moving wall has a negative one at  $y = R(t)$ . For  $k < 0$ , the situation is opposite. If  $kv > 0$ , the scale factor on the moving wall is exponentially contracted as  $\tau$  runs. This is because the wall is moving toward the inside the AdS space, where the scale factor decreases. If  $kv < 0$ , the wall is moving toward the boundary: the scale factor increases and this is described as an inflation by the the observer at the domain wall.

When we consider the region  $|y| \geq R$ , we have similar situation but with just one wall at  $y = R(t)$ . Exactly parallel discussion can be done. however this is the case which is already discussed in ref. [4]. So we so not treat it here.

## B. Moving domain walls with all positive tension

In the previous case, we imposed  $\mathbf{Z}_2$  symmetry so that we glued two vacuum AdS solutions bounded by one static and one moving brane whose velocity is determined by its tension. We also noticed that one of the brane tension must be negative. If we want to have solution with two or more branes having the same sign, we can not in general impose  $\mathbf{Z}_2$  symmetry and at least one side of the wall must be AdS-Schwarzschild solution (AdSS). The situation is just like the case of spherical shells in the Minkowski space: Inside the shell, it is vacuum solution while it must be Schwarzschild solution outside. The motion of each shell can be is described by the Israel junction condition. More explicitly, Let there be  $N$  walls at  $y = y_i(t)$   $i = 1, 2, \dots, N$ . Now suppose that the metric in the region  $y_i < y < y_{i+1}$  is described by the AdSS:

$$ds^2 = e^{2ky} \left[ -(1 - \mu_i e^{-4ky}) dt^2 + d\vec{x}^2 \right] + \frac{dy^2}{1 - \mu_i e^{-4ky}}. \quad (40)$$

Then the equation of motion for the wall is given by

$$\dot{y}_i^2 + 1 = \left( \frac{\sigma}{\sigma_c} \right)^2 + \frac{1}{4} (\mu_i + \mu_{i-1}) e^{-4ky_i} + \frac{1}{32} \left( \frac{\sigma_c}{\sigma} \right)^2 (\mu_i - \mu_{i-1})^2 e^{-8ky_i}. \quad (41)$$

What is important for us at this moment is just the existence of such solution describing the multi-domain walls with all positive tension.

### C. Cosmology with Moving Domain wall

Now let's try to realize a situation where there is a static domain wall at  $y = y_1 = 0$  and there is a moving brane at  $y = y_2(t) > 0$ . In the leftmost region  $y < 0$  we have a AdS vacuum. In between the two walls, we have a AdSS solution with non-extremal parameter  $\mu_1$ . In the right most region  $y > y_2(t)$ , we have another AdSS solution with parameter  $\mu_2$ . The motion for the first wall is described by

$$\dot{y}_1^2 + 1 = \left(\frac{\sigma}{\sigma_c}\right)^2 + \frac{1}{4}(\mu_1)e^{-4ky_1} + \frac{1}{32}\left(\frac{\sigma_c}{\sigma}\right)^2 (\mu_1)^2 e^{-8ky_1}. \quad (42)$$

If we want to have a static wall at  $y = 0$ ,

$$\sigma = \sigma_c \sqrt{\frac{1}{2}(1 - \mu_1/4) + \frac{1}{2}\sqrt{1 - \frac{5}{8}\mu_1 + \frac{1}{16}\mu_1^2}} \quad (43)$$

This is the condition for the static wall.

Another fact one should notice is that in the absence of the orbifold symmetry, the effective 4 dimensional cosmological constant on the brane is not well defined. Decomposing the 5 dimensional Einstein equation to the paralell and vertical to the brane, we get 4 dimensional equation which contains term proportional to the induced metric [8] to be identified as the 4 dimensional cosmological constant. It is a sum of bulk cosmological constant and a quadratic form of the extrinsic curvature:

$$\Lambda_4 = \frac{\kappa^2}{2}\Lambda_5 + \frac{1}{2}((K_\mu^\mu)^2 - K^{\mu\nu}K_{\mu\nu}) + \frac{1}{4}(K_m^\sigma K_{n\sigma}g^{mn} - K_\mu^\mu K_{mn}g^{mn}), \quad (44)$$

where  $K$  is the extrinsic curvature tensor and greek indices are 5 dimensional indices while roman ones are 4 dimensional. In the presence of the orbifold symmetry, the extrinsic curvatures across the brane,  $K_{\mu\nu}^+$  and  $K_{\mu\nu}^-$ , differ only in sign, which is indifferent in the quadratic form. In the absence of the symmetry, there is no way to define the effective 4

dimensional constant naturally. The best thing one can do is do define by hand such that the stationary brane has zero effective cosmological constant. We will not push this idea further here. The motion of the second wall is described by (41) with two parameter  $\mu_1, \mu_2$ .

What happens to the stationary brane(our universe) if the other brane comes and overlaps? Even in the absence of the interaction between the branes, tensions will be added therefore we expect that the stationary brane must move and inflate. If both branes are infinitely thin, the impact is instantaneous and the consequence is expected to be small if we neglect the interaction of the two branes. If the impact brane is thick, then overlapping will change the effective bulk cosmological constant near the thin brane therefore cause a motion of the stationary brane. (Remember that the static condition is the balance of the bulk cosmological constant and the brane tension both with and without the orbifold symmetry.) During the scattering, the thin brane moves hence inflates. After the thick brane pass away, the condition of the static brane is restored and it must stop moving. Therefore the process of thin and thick brane scattering gives a mechanism of inflation for finite time. In other words, there is a natural graceful exit out of the inflation [9]. Furthermore if the moving brane is thick enough, the scattering process is smooth enough so that the density fluctuation caused by the inflation may fit the present observation.

#### IV. CONCLUSION

We considered moving-brane-solutions in AdS type background. In the first Randall-Sundrum configuration, there are two branes at fixed points of the orbifold symmetry. We proved that that if one brane is fixed and the other brane is moving, the configuration is still a solution provided that the motion of the brane has a specific velocity determined by its tension and the bulk cosmological constant. In the absence of the  $\mathbf{Z}_2$  symmetry, we could construct multi-brane configurations patching AdS-Schwarzschild solutions. In this case, effective four dimensional cosmological constant on the brane is ambiguous. Instead, we found a condition for a brane to be stationary. Finally, we suggest a scenario where we

may have inflation on the brane universe for finite duration of time, i.e, a graceful exit of inflation. Our description of the last point was very qualitative and also did not consider important fact: two branes must interact at least gravitationally. We also did not study the consequence of the RS1 due to the brane motion. We wish to comeback to treatment of these aspects in future publication.

### **Acknowledgments**

SJS want to thank S. Nam and P. Yi for discussion. He also want to express thank KIAS for its hospitality during his visit. This work has been supported by KOSEF 1999-2-112-003-5 and BK21.

### **Note added**

After typing of this manuscript was finished, we saw the appearance of the paper hep-th/0004206 by Horowitz, Low and Zee, where a wave solution were constructed in a similar fashion.

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